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SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ in covariant baryon chiral perturbation theory

L. S. Geng*
Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, E-46071 Valencia, Spain
E-mail: lsgeng@ific.uv.es

J. Martin Camalich
Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, E-46071 Valencia, Spain
E-mail: camalich@ific.uv.es

M.J. Vicente Vacas
Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, E-46071 Valencia, Spain
E-mail: vicente@ific.uv.es

We report on a recent study of the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ up to $\mathcal{O}(p^4)$ in covariant baryon chiral perturbation theory with dynamical octet and decuplet contributions. The decuplet contributions are taken into account for the first time in a covariant ChPT study and are found of similar or even larger size than the octet ones. We predict positive SU(3)-breaking corrections to all the four independent $f_1(0)$'s (assuming isospin symmetry), which are consistent, within uncertainties, with the latest results from large N_c fits, chiral quark models, and quenched lattice QCD calculations. We also discuss briefly the implications of our results for the extraction of V_{us} from hyperon decay data.

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*Speaker.

1. Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2]

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1.1}$$

plays a very important role in our study and understanding of flavor physics. Particularly, an accurate value of V_{us} is crucial in determinations of the other parameters and in tests of CKM unitarity. For instance, to test the first row unitarity,

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1, \tag{1.2}$$

one needs to know the values of V_{ud}, V_{us}, and V_{ub}. Among them, V_{ub} is quite small and can be neglected at the present precision; V_{ud} can be obtained from superallowed nuclear beta decays, neutron and pion decays; while V_{us} can be obtained from kaon decays, hyperon decays, and tau decays (for a recent review, see Ref. [3]). In this work, we focus on an important quantity in order to obtain V_{us} from hyperon decay data – the $f_1(0)$.

To extract V_{us} from hyperon decay data, one must know the hyperon vector coupling $f_1(0)$, since experimentally only the product of $|V_{us} f_1(0)|$ is accessible. Theoretically, $f_1(0)$ is known up to SU(3) breaking corrections due to the hypothesis of Conservation of Vector Current (CVC) . To obtain an accurate $f_1(0)$, one then needs to know the size of SU(3) breaking, which, naively, could be as large as 30%. On the other hand, the Ademollo-Gatto (AG) theorem [4] states that

$$f_1(0) = g_v + \mathscr{O}((m_s - m)^2) \tag{1.3}$$

where m_s is the strange quark mass and m is the mass of the light quarks. The values of g_V are $-\sqrt{\frac{3}{2}}$, $-\frac{1}{\sqrt{2}}$, -1, $\sqrt{\frac{3}{2}}$, $\frac{1}{\sqrt{2}}$, 1 for $\Lambda \to p$, $\Sigma^0 \to p$, $\Sigma^- \to n$, $\Xi^- \to \Lambda$, $\Xi^- \to \Sigma^0$, and $\Xi^0 \to \Sigma^+$, respectively. In the isospin-symmetric limit only four channels, which we take as $\Lambda \to N$, $\Sigma \to N$, $\Xi \to \Lambda$, and $\Xi \to \Sigma$, provide independent information. The AG theorem not only tells that SU(3)-breaking corrections are of the order of 10% but also has a very important consequence for a calculation of $f_1(0)$ in chiral perturbation theory as we will explain below.

Theoretical estimates of SU(3)-breaking corrections to $f_1(0)$ have been performed in various frameworks, including quark models [5, 6, 7], large-N_c fits [8], and chiral perturbation theory (ChPT) [9, 10, 11, 12, 13]. These SU(3)-breaking corrections have also been studied recently in quenched lattice QCD (LQCD) calculations for the two channels: $\Sigma^- \to n$ [14] and $\Xi^0 \to \Sigma^+$ [15].

Compared to earlier ChPT studies [9, 10, 11, 12, 13], our work [16] contains two improvements:

1. We have performed a calculation that fully conserves analyticity and relativity without introducing any power-counting-restoration (PCR) dependence. This is possible because the Ademollo-Gatto theorem tells that up to $\mathscr{O}(p^4)$ no unknown LEC's contribute and, therefore, no power-counting-breaking terms shows up. Consequently, up to this order there is no need to apply any PCR procedure.

2. We have taken into account the contributions of virtual decuplet baryons. They are important because $m_D - m_B \approx 0.231$ GeV is similar to the pion mass and much smaller than the kaon (eta) mass, where m_D and m_B are the averages of the octet- and decuplet-baryon masses, respectively. Therefore, in SU(3) baryon ChPT, one has to be cautious about the exclusion of virtual decuplet baryons. As we will show, the decuplet baryons do provide sizable contributions that completely change the results obtained with only virtual octet baryons

2. Formalism

The baryon vector form factors as probed by the charged ΔS=1 weak current $V^\mu = V_{us}\bar{u}\gamma^\mu s$ are defined by

$$\langle B'|V^\mu|B\rangle = V_{us}\bar{u}(p')\left[\gamma^\mu f_1(q^2) + \frac{2i\sigma^{\mu\nu}q_\nu}{M_{B'}+M_B} f_2(q^2) + \frac{2q^\mu}{M_{B'}+M_B} f_3(q^2)\right] u(p), \tag{2.1}$$

where $q = p' - p$. We will parameterize the SU(3)-breaking corrections order-by-order in the covariant chiral expansion as follows:

$$f_1(0) = g_V\left(1 + \delta^{(2)} + \delta^{(3)} + \cdots\right), \tag{2.2}$$

where $\delta^{(2)}$ and $\delta^{(3)}$ are the leading and next-to-leading order SU(3)-breaking corrections induced by loops, corresponding to $\mathscr{O}(p^3)$ and $\mathscr{O}(p^4)$ chiral calculations.

The calculation of the virtual octet-baryon contributions is standard and details can be found in Ref. [16]. Here we would like to stress the calculation of the virtual decuplet-baryon contributions. A fully consistent and problem-free description of spin-3/2 particles in a quantum-field-theory framework is not yet possible, although progress has been made in the past few decades (see e.g. Refs. [17, 18]). In the framework of effective field theories, such as ChPT, a solution has been proposed in Ref. [17], where the couplings of spin-3/2 particles to spin-1/2 particles satisfy spin-3/2 gauge symmetry and are referred to "consistent" couplings. In addition to this formal appealing nature, certain ChPT calculations performed with these consistent couplings have shown better convergence behavior than the same calculations done in the "conventional coupling" scheme (see e.g. Refs. [19]). In the present work, we have adopted the "consistent" couplings to describe the interactions of decuplet-baryons with octet-baryons. Details can be found in Ref. [16].

3. Results and discussions

3.1 SU(3)-breaking corrections to $f_1(0)$ due to octet contributions up to $\mathscr{O}(p^4)$

All the diagrams contributing to $f_1(0)$ up to $\mathscr{O}(p^4)$ are shown in Fig. 1, where the leading and next-to-leading order SU(3)-breaking corrections are given by the diagrams in the first and second row, respectively.

The $\mathscr{O}(p^3)$ results are quite compact and have the following structure for the transition $i \to j$:

$$\delta_B^{(2)}(i \to j) = \sum_{M=\pi,\eta,K} \beta_M^{\mathrm{BP}} H_{\mathrm{BP}}(m_M) + \sum_{M=\pi,\eta} \beta_M^{\mathrm{MP}} H_{\mathrm{MP}}(m_M, m_K) + \sum_{M=\pi,\eta,K} \beta_M^{\mathrm{KR}} H_{\mathrm{KR}}(m_M)$$

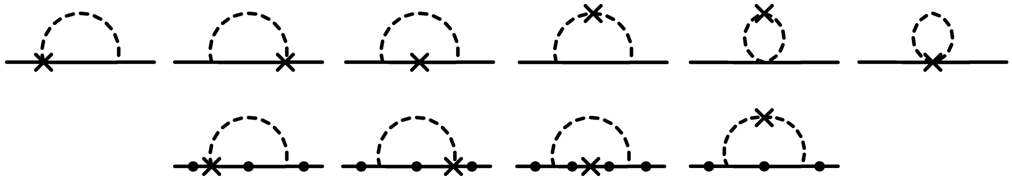

Figure 1: Feynman diagrams contributing to the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ up to $\mathscr{O}(p^4)$. The solid lines correspond to baryons and dashed lines to mesons; crosses indicate the coupling of the external current; black dots denote mass splitting insertions. We have not shown explicitly those diagrams corresponding to wave function renormalization, which have been taken into account in the calculation.

$$
\begin{aligned}
&-\frac{3}{8}\sum_{M=\pi,\eta} H_{\mathrm{TD1}}(m_M,m_K)+\frac{3}{8}\sum_{M=\pi,\eta} H_{\mathrm{TD2}}(m_M)+\frac{3}{4}H_{\mathrm{TD2}}(m_K)\\
&+\frac{1}{2}\sum_{M=\pi,\eta,K}(\beta_M^{\mathrm{WF}}(i)+\beta_M^{\mathrm{WF}}(j))H_{\mathrm{WF}}(m_M),
\end{aligned}
\tag{3.1}
$$

where β^{BP}, β^{MP}, β^{KR}, and β^{WF} , and the loop functions H_{BP}, H_{MP}, H_{KR}, H_{TD1}, H_{TD2}, and H_{WF} are given in Appendix A of Ref. [16]. It is interesting to point out that although separately these loop functions are divergent (scale-dependent) and some of them contain power-counting-breaking pieces (H_{KR} and H_{MP}), the overall contributions are finite and do not break power-counting. This is an explicit manifestation of the AG theorem.

Similar to the IRChPT study of Ref. [13], the $\mathscr{O}(p^4)$ results contain higher-order divergences. We have removed the infinities using the modified minimal-subtraction ($\overline{MS}$) scheme. The remaining scale dependence is shown in Fig. 2 of Ref. [16], which is rather mild for most cases except for the $\Sigma \to N$ transition. The scale dependence can also be used to estimate the size of higher-order contributions by varying μ in a reasonable range. In the following, we present the results by varying μ from 0.7 to 1.3 GeV. It should be mentioned that if we had adopted the same method as Ref. [12] to calculate the $\mathscr{O}(p^4)$ contributions, i.e., by expanding the results and keeping only those linear in baryon mass splittings, our $\mathscr{O}(p^4)$ results would have been convergent.

Table 2 shows the SU(3)-breaking corrections in the notation of Eq. (2.2). For the sake of comparison, we also list the numbers obtained in HBChPT [12] and IRChPT [13]. The numerical values are obtained with the parameters given in Table 1. As in Ref. [20] we have used an average $F_0 = 1.17 f_\pi$ with $f_\pi = 92.4$ MeV. It should be pointed out that the HBChPT and the IRChPT results are obtained using f_π.

First, we note that in three of the four cases, the $\delta^{(3)}$ numbers are smaller than the $\delta^{(2)}$ ones. The situation is similar in IRChPT but quite different in HBChPT. In the HBChPT calculation [12], the $\delta^{(3)}$ contribution is larger than the $\delta^{(2)}$ one for all the four transitions.[1] On the other hand, the results of the present work and those of IRChPT [13], including the contributions of different chiral orders, are qualitatively similar. They are both very different from the HBChPT predictions, even for the signs in three of the four cases. Obviously, as stressed in Ref. [13], one should trust more the covariant than the HB results, which have to be treated with caution whenever $1/M$ recoil corrections become large, as in the present case [12].

[1]What we denote by $\delta^{(3)}$ is the sum of those labeled by $\alpha^{(3)}$ and $\alpha^{(1/M)}$ in Ref. [12].

Table 1: Values for the masses and couplings appearing in the calculation of the SU(3)-breaking corrections to $f_1(0)$.

D	0.8	M_B	1.151 GeV
F	0.46	M_D	1.382 GeV
f_π	0.0924 GeV	M_0	1.197 GeV
F_0	$1.17 f_\pi$	b_D	-0.0661 GeV^{-1}
m_π	0.138 GeV	b_F	0.2087 GeV^{-1}
m_K	0.496 GeV	M_{D0}	1.216 GeV
m_η	0.548 GeV	γ_M	0.3236 GeV^{-1}
$\mathscr{C}$	1.0		

Table 2: Octet contributions to the SU(3)-breaking corrections to $f_1(0)$ (in percentage). The central values of the $\mathscr{O}(p^4)$ results are calculated with $\mu = 1$ GeV and the uncertainties are obtained by varying μ from 0.7 to 1.3 GeV.

	present work			HBChPT [12]	IRChPT [13]
	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$
$\Lambda \to N$	-3.8	$0.2^{+1.2}_{-0.9}$	$-3.6^{+1.2}_{-0.9}$	2.7	-5.7 ± 2.1
$\Sigma \to N$	-0.8	$4.7^{+3.8}_{-2.8}$	$3.9^{+3.8}_{-2.8}$	4.1	2.8 ± 0.2
$\Xi \to \Lambda$	-2.9	$1.7^{+2.4}_{-1.8}$	$-1.2^{+2.4}_{-1.8}$	4.3	-1.1 ± 1.7
$\Xi \to \Sigma$	-3.7	$-1.3^{+0.3}_{-0.2}$	$-5.0^{+0.3}_{-0.2}$	0.9	-5.6 ± 1.6

3.2 SU(3)-breaking corrections to $f_1(0)$ induced by dynamical decuplet baryons up to $\mathscr{O}(p^4)$

Fig. 2 shows the diagrams that contribute to SU(3)-breaking corrections to $f_1(0)$ with dynamical decuplet baryons up to $\mathscr{O}(p^4)$. It should be noted that unlike in the HBChPT case [12], Kroll-Rudermann (KR) kind of diagrams also contribute. In fact, using the consistent coupling scheme of Ref. [17], there are four KR diagrams: Two are from minimal substitution in the derivative of the pseudoscalar fields and the other two are from minimal substitution in the derivative of the decuplet fields.

As in the previous case, the $\mathscr{O}(p^4)$ results contain again higher-order divergences, which have been removed by the $\overline{MS}$ procedure with the remaining scale dependence shown in Fig. 4 of Ref. [16]. In this case, unlike in the previous case, the divergences cannot be removed by expanding and keeping only terms linear in baryon and decuplet mass splittings.

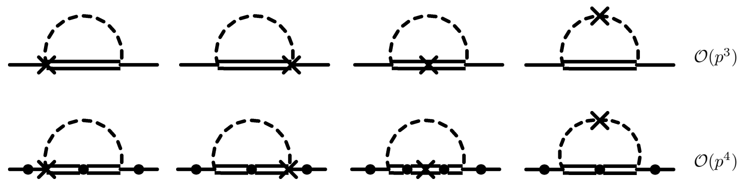

Figure 2: Feynman diagrams contributing to the leading and next-to-leading order SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$, through dynamical decuplet baryons. The notations are the same as those of Fig. 1 except that double lines indicate decuplet baryons. We have not shown explicitly those diagrams corresponding to wave function renormalization, which have been included in the calculation.

Table 3: Decuplet contributions to the SU(3)-breaking corrections to $f_1(0)$ (in percentage). The central values of the $\mathcal{O}(p^4)$ result are calculated with $\mu = 1$ GeV and the uncertainties are obtained by varying μ from 0.7 to 1.3 GeV.

	Present work			HBChPT		
	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$
$\Lambda \to N$	0.7	$3.0^{+0.1}_{-0.1}$	$3.7^{+0.1}_{-0.1}$	1.8	1.3	3.1
$\Sigma \to N$	-1.4	$6.2^{+0.4}_{-0.3}$	$4.8^{+0.4}_{-0.3}$	-3.6	8.8	5.2
$\Xi \to \Lambda$	-0.02	$5.2^{+0.4}_{-0.3}$	$5.2^{+0.4}_{-0.3}$	-0.05	4.2	4.1
$\Xi \to \Sigma$	0.7	$6.0^{+1.9}_{-1.4}$	$6.7^{+1.9}_{-1.4}$	1.9	-0.2	1.7

The numerical results obtained with the parameter values given in Table 1 are summarized in Table 3. It can be seen that at $\mathcal{O}(p^3)$, the decuplet contributions are relatively small compared to the octet ones at the same order. On the other hand, the $\mathcal{O}(p^4)$ contributions are sizable and all of them have positive signs.

In Table 3, the numbers denoted by HBChPT are obtained by taking our covariant results to the heavy-baryon limit, as explained in detail in Ref. [16]. They are different from those of Ref. [12] and the origin of the difference is also discussed in Ref. [16].

3.3 Full results and comparison with other approaches

Summing the octet and the decuplet contributions, we obtain the numbers shown in Table 4. Two things are noteworthy. First, the convergence is slow, even taking into account the scale dependence of the $\delta^{(3)}$ corrections. Second, for three of the four transitions, the $\delta^{(3)}$ corrections have a different sign than the $\delta^{(2)}$ ones.

In Table 5, we compare our results with those obtained from other approaches, including large N_c fits [8], quark models [5, 6, 7], and two quenched LQCD calculations [14, 15]. The large N_c results in general favor positive corrections, which are consistent with our central values. Two of the quark models predict negative corrections, while that of Ref. [7] favors positive corrections. It is interesting to note that in Ref. [7] the valence quark effects give negative contributions, as in the other two quark models; on the other hand, the chiral effects provide positive contributions, result-

Table 4: SU(3)-breaking corrections to $f_1(0)$ up to $\mathcal{O}(p^4)$ (in percentage), including both the octet and the decuplet contributions.

	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$
$\Lambda\to N$	-3.1	$3.2^{+1.3}_{-1.0}$	$0.1^{+1.3}_{-1.0}$
$\Sigma\to N$	-2.2	$10.9^{+4.2}_{-3.1}$	$8.7^{+4.2}_{-3.1}$
$\Xi\to\Lambda$	-2.9	$6.9^{+2.8}_{-2.1}$	$4.0^{+2.8}_{-2.1}$
$\Xi\to\Sigma$	-3.0	$4.7^{+2.2}_{-1.6}$	$1.7^{+2.2}_{-1.6}$

Table 5: SU(3)-breaking corrections (in percentage) to $f_1(0)$ obtained in different approaches.

	Present work	Large N_c	Quark model			Quenched LQCD
		Ref. [8]	Ref. [5]	Ref. [6]	Ref. [7]	
$\Lambda\to N$	$0.1^{+1.3}_{-1.0}$	2 ± 2	-1.3	-2.4	0.1	
$\Sigma\to N$	$8.7^{+4.2}_{-3.1}$	4 ± 3	-1.3	-2.4	0.9	$-1.2\pm2.9\pm4.0$ [14]
$\Xi\to\Lambda$	$4.0^{+2.8}_{-2.1}$	4 ± 4	-1.3	-2.4	2.2	
$\Xi\to\Sigma$	$1.7^{+2.2}_{-1.6}$	8 ± 5	-1.3	-2.4	4.2	-1.3 ± 1.9 [15]

ing in net positive corrections. Our numbers also agree, within uncertainties, with the quenched LQCD ones.

3.4 Implications for the extraction of V_{us} from hyperon decay data

In the following, we briefly discuss the implications of our results for the extracting of V_{us} from hyperon decay data. There have been several previous attempts to extract V_{us} using hyperon semileptonic decay data [21, 22, 8, 23]. As discussed in Ref. [23] a rather clean determination of f_1V_{us} can be done by using g_1/f_1 and the decay rates from experiment and taking for g_2 and f_2 their SU(3) values. This latter approximation is supported by the fact that their contributions to the decay rate are reduced by kinematic factors (See, for instance, Eq. (10) of Ref. [8]). Using the values of f_1V_{us} compiled in Table 3 of Ref. [23] and our results for f_1 we get

$$V_{us}=0.2177\pm0.0030, \tag{3.2}$$

where the error includes only the experimental errors and the uncertainties related to the scale dependence. This value is lower than the value obtained in Refs. [21, 22], which is easy to understand because our procedure is similar to that of Refs. [21, 22] but our calculated SU(3) breaking corrections are positive while they are assumed to be zero in Refs. [21, 22].

4. Summary and conclusions

The CKM matrix element V_{us} plays a very important role in studies of flavor physics. At present, its most accurate value is obtained from kaon decays. However, it will be of vital importance to be able to extract its value also from other sources, such as hyperon decay data.

To extract V_{us} from hyperon decay data, one must know very accurately the value of $f_1(0)$ since experimentally only the product of $V_{us} f_1(0)$ is accessible. Chiral perturbation theory provides a model-independent prediction for $f_1(0)$ up to $\mathcal{O}(p^4)$, thanks to the Ademollo-Gatto theorem.

In this work, we have performed a fully-covariant calculation up to $\mathcal{O}(p^4)$ in chiral perturbation theory taking into account the contributions of virtual decuplet baryons, which are found to be comparable in size to the virtual octet-baryon contributions. Our study predicts positive SU(3) breaking corrections to $f_1(0)$ in all the four channels, $\Lambda \to N$, $\Sigma \to N$, $\Xi \to \Lambda$, and $\Xi \to \Sigma$, in agreement with those obtained from the large N_c fits. We encourage the use of our calculated $f_1(0)$ in future analysis of hyperon decay data.

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